CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define dot product between two vectors $\vec{a}$ and $\vec{b}$. Find the sine of the angle between $\vec{a} = 4\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} - 2\vec{k}$. **(08 Marks)**

b. Express $\dfrac{3+4i}{3-4i}$ in the form of $x + iy$ and hence find its modulus and amplitude. **(06 Marks)**

c. Find the real part of $\dfrac{1}{1+\cos\theta + i\sin\theta}$. **(06 Marks)**

OR

2 a. Prove that $\left[\dfrac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right]^4 = \cos 8\theta + i\sin 8\theta$ **(08 Marks)**

b. If $\vec{A} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 2\vec{j} + \vec{k}$. Show that $\left(\vec{A} + \vec{B}\right)$ and $\left(\vec{A} - \vec{B}\right)$ are orthogonal. **(06 Marks)**

c. Find the value of λ so that the vectors $\vec{A} = 3\vec{i} + 5\vec{j} - 3\vec{k}$, $\vec{B} = \vec{i} + \lambda\vec{j} + 2\vec{k}$ and $\vec{C} = 2\vec{i} - 2\vec{j} + \vec{k}$ are co-planar. **(06 Marks)**

Module-2

3 a. If $y = \tan^{-1}x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(08 Marks)**

b. Find the angle between the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(06 Marks)**

c. If $u = \tan^{-1}\left[\dfrac{x^3+y^3}{x-y}\right]$. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. **(06 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of $\tan x$ upto the term containing x^5. **(08 Marks)**

b. Find the Pedal equation to the curve $r^m = a^m \cos m\theta$. **(06 Marks)**

c. If $u = f(x-y, y-z, z-x)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\displaystyle\int_0^{\pi/2} \cos^n x\,dx$ (n >0). **(08 Marks)**

b. Evaluate $\displaystyle\int_0^a x\sqrt{ax - x^2}\,dx$. **(06 Marks)**

c. Evaluate $\displaystyle\iint_R xy\,dx\,dy$ where R is the I quadrant of the circle $x^2 + y^2 = a^2$, $x \geq 0$, $y \geq 0$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$ (n >0). **(08 Marks)**

b. Using reduction formula evaluate $\int_0^1 x^5 (1-x^2)^{\frac{5}{2}} dx$. **(06 Marks)**

c. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz dx dy dz$. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x=(1-t^3)$, $y=(1+t^2)$ and $z=(2t-5)$. Determine the components of velocity and acceleration at t = 1 in the direction $2i+j+2k$. **(08 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2i-j-2k$. **(06 Marks)**

c. If $\vec{F} = (x+y+1)i + j - (x+y)k$, show that $\vec{F} \cdot \text{curl} \vec{F} = 0$ **(06 Marks)**

OR

8 a. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$ **(08 Marks)**

b. Show that $\vec{F} = \dfrac{xi+yj}{x^2+y^2}$ is solenoidal. **(06 Marks)**

c. Find the values of the constants a, b, c such that $\vec{F} = (x+y+az)i + (6x+2y-z)j + (x+cy+2z)k$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve : $(1+y^2)dx = (\tan^{-1} y - x)\, dy$. **(08 Marks)**

b. Solve : $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. **(06 Marks)**

c. Solve : $\left[x \tan\left(\dfrac{y}{x}\right) - y\sec^2\left(\dfrac{y}{x}\right)\right]dx + x\sec^2\left(\dfrac{y}{x}\right)dy = 0$. **(06 Marks)**

OR

10 a. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$. **(08 Marks)**

b. Solve : $y(x+y)dx + (x+2y-1)dy = 0$. **(06 Marks)**

c. Solve : $\left[y^2 e^{xy^2} + 4x^3\right]dx + \left[2xye^{xy^2} - 3y^2\right] dy = 0$ **(06 Marks)**

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